GCE AS MARKING SCHEME

SUMMER 2024

AS
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 2 FURTHER STATISTICS A

SUMMER 2024 MARK SCHEME

Qu.	Solution	Mark	Notes
1(a)(i)	Total number of fish caught, F, is $\text{Po}((3.8 + 4.3) \times 0.5)$ $\text{Po}(4.05)$	M1 M1	M1 for Poisson (si) and adding. M1 for multiplying by 0.5, oe. M0M1 for 4.05 with Poisson not mentioned nor used. M0M1 for $\text{Po}(2.15)$ or $\text{Po}(1.9)$
	$\text{P}(F < 2) = \frac{4.05^1 \times e^{-4.05}}{1!} + \frac{4.05^0 \times e^{-4.05}}{0!}$	M1	Use of formula or calculator with their $\lambda \neq 3.8$ or 4.3
	$= 0.08798$	A1	cao Condone 0.088
	ALTERNATIVE SOLUTION Total number of fish caught by Dave, D, is $\text{Po}(4.3 \times 0.5)$ $\text{Po}(2.15)$	(M1)	M1 for both Dave and Llinos
	Total number of fish caught by Llinos, L, is $\text{Po}(3.8 \times 0.5)$ $\text{Po}(1.9)$ Possible combinations are $D = 0$ and $L = 0$ $D = 0$ and $L = 1$ OR $D = 1$ and $L = 0$		
	$\text{P}(D = 0 \text{ and } L = 0) = \frac{2.15^0 \times e^{-2.15}}{0!} \times \frac{1.9^0 \times e^{-1.9}}{0!}$ $\text{P}(D = 0 \text{ and } L = 1) = \frac{2.15^0 \times e^{-2.15}}{0!} \times \frac{1.9^1 \times e^{-1.9}}{1!}$ $\text{P}(D = 1 \text{ and } L = 0) = \frac{2.15^1 \times e^{-2.15}}{1!} \times \frac{1.9^0 \times e^{-1.9}}{0!}$	(M1)	M1 for use of formula once.
	$\text{P}(F < 2) = 0.017422 + 0.033102 + 0.037458$ $\text{P}(F < 2) = 0.08798$	(M1) (A1)	M1 for addition
(ii)	Valid justification in context. e.g. Fish are caught singly. Fish are caught independently. Catches occur at random. Dave and Llinos catch fish independently. Constant average rate of fish being caught.	E1 **[5]**	Must be in context E0 for constant rate

Qu.	Solution	Mark	Notes
1(b)(i)	Expected time until next fish $= \frac{1}{4.3} \times 8$	M1	oe
	1.86 hours or 1 hour 52 minutes		
	i.e. 12.52pm	A1	Accept 12.51pm A0 for 1.86 only
(ii)	P (Dave doesn't catch a fish for the rest of the day) $= \mathrm{e}^{-4.3\times 0.5}$	M1	
	$= 0.1165$	A1	
	<u>Alternative solution</u> Using $\lambda = 2.15$ AND $\mathrm{P}(X = 0)$	(M1)	si
	$\mathrm{P}(X = 0) = 0.1165$	(A1)	
		[4]	
(c)	Let J be the number of trout she catches in a year. $J \sim \mathrm{B}(950, 0.02)$	B1	si
	$\mathrm{P}(J \geq 30) = 1 - \mathrm{P}(J \leq 29)$ $= 0.01109$	M1 A1	
		[3]	
(d)	$\mathrm{Po}(19)$ Poisson since n is large and p is small.	B1 E1	Condone similar with values e.g. $n > 50$, $p < 0.1$ e.g. $np > 10$
		[2]	
	Total for Question 1	**14**	

Qu.	Solution	Mark	Notes
2(a)	Realising Q_3 is in the third part of the CDF.	B1	si
	$\frac{x^2 - x + 3}{5} = 0.75$	M1	Setting $F(x) = 0.75$ Allow $\frac{x+2}{5} = 0.75$ for M1 only
	$x^2 - x - 0.75 = 0$	A1	oe
	$x = -0.5$ or $x = 1.5$	A1	Both values.
	Reject $-0.5 \therefore x = 1.5$	B1 **[5]**	FT provided quadratic, with one answer in the range [1,2] and one outside this range.
(b)	$f(x) = F'(x)$	M1	M1 Attempt at differentiating with at least one power of x decreasing.
	$f(x) = \begin{cases} \frac{1}{5} & -2 \le x < 1 \\ \frac{2x-1}{5} & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$	A1 A1 B1 **[4]**	A1 Correct expression for $f(x)$ for $-2 \le x < 1$. A1 Correct expression for $f(x)$ for $1 \le x \le 2$. B1 for "0 otherwise" and correct ranges.
(c)(i)	$\mathrm{E}(X) = \int_{-2}^{1} \frac{x}{5} \mathrm{d}x + \int_{1}^{2} \frac{2x^2 - x}{5} \mathrm{d}x$	M1	M1 Attempt at integrating $xf(x)$ with at least one power of x increasing (ignore limits here)
	$\mathrm{E}(X) = \left[\frac{x^2}{10}\right]_{-2}^{1} + \left[\frac{2x^3}{15} - \frac{x^2}{10}\right]_{1}^{2}$	A1	A1 correct integration with correct limits. FT 'their $f(x)$' of equivalent difficulty
	$\mathrm{E}(X) = \frac{1}{3}$ (minutes)	A1	cao
(ii)	Valid interpretation e.g. 20 seconds longer than the target time **on average**.	E1 **[4]**	FT their $\mathrm{E}(X)$
	Total for Question 2	**13**	

Qu.	Solution	Mark	Notes
3(a)	H_0:The number of bags sold can be modelled by a Poisson distribution with mean 2.2. H_1:The number of bags sold cannot be modelled by a Poisson distribution with mean 2.2.	B1	
	No. bags sold: 0, 1, 2, 3, 4, 5+ Exp Freq: 5.54, 12.19, 13.41, 9.83, 5.41, 3.62	B1 B1	B1 for 2 values correct. B1 All correct.
	Combining classes No. bags sold: 0, 1, 2, 3, 4+ Exp Freq: 5.54, 12.19, 13.41, 9.83, 9.03	M1	M1 combining classes.
	Use of χ^2 test stat $=\sum\frac{(O-E)^2}{E}$.	M1	SC for solution that does not combine classes. (M0M1m1A0B1B1m1A0)
	$=\frac{(7-5.54)^2}{5.54}+\frac{(10-12.19)^2}{12.19}+\frac{(11-13.41)^2}{13.41}+\frac{(9-9.83)^2}{9.83}+\frac{(13-9.03)^2}{9.03}$	m1	Alternative M1m1 $\frac{7^2}{5.54}+\frac{10^2}{12.19}+\frac{11^2}{13.41}+\frac{9^2}{9.83}+\frac{13^2}{9.03}-50$
	$= 3.02\ldots$	A1	cao Accept 3.03 Accept 3.016…
	DF = 4	B1	
	10% CV = 7.779	B1	
	Since $3.02\ldots < 7.779$ there is insufficient evidence to Reject H_0.	m1	Dependent on 2nd M1 FT their TS and CV
	There is no evidence to suggest that the number of bags sold cannot be modelled by a Poisson (2.2) distribution.	A1 **[11]**	cso
3(b)	$\mathrm{Po}(2.5)$ is better since the χ^2 test statistic is smaller.	E1 **[1]**	FT their TS from (a)
	Total for Question 3	**12**	

Qu.	Solution	Mark	Notes
4 (a)	$r = \dfrac{-18895.13043}{\sqrt{11503.91304 \times 78669.30435}}$	M1	Condone +ve numerator
	$r = -0.628$	A1	3sf
	Valid interpretation. e.g. The negative value of 0.628 implies that on average the more people rely on cash, the less debt they have as a proportion of their household income. e.g. The correlation is fairly strong between household debt and the use of cash for transactions. e.g. As the percentage of cash transactions increases, net household debt as a percentage of disposable income tends to decrease. e.g. Household debt and percentage of cash transactions are negatively correlated.	E1 **[3]**	
(b)	$b = \frac{S_{xy}}{S_{xx}}$	M1	
	$b = -1.64$	A1	
	$a = \dfrac{2695}{23} + 1.6424959\ldots \times \dfrac{1467}{23}$	M1	FT their b for M1A0
	$a = 221.9\ldots$	A1	
	$y = 222 - 1.64x$	B1	Correct to 3sf, ft their a and b
		[5]	
(c)	Selecting correct equation to use in each case.		FT (b) for Malta
	Malta $p = 222 - 1.64 \times 92$ $p = 71.12$	B1	Accept anything rounding to 71
	Denmark $q = -0.24 \times 261 + 91.92$ $q = 29.28$	B1 **[2]**	
(d)	Valid explanation e.g. Not reliable because it's extrapolation, rather than interpolation. e.g. May not be reliable because the values are very close to the extremes of the graph. e.g. May be reliable because there is fairly strong correlation and the values are only just outside the range of the graph.	E1	
(e)	Valid explanation. e.g. Net disposable income may be negative. e.g. Household incomes are less than outgoings leading to a negative value for net disposable income. e.g. Maybe an erroneous value.	E1 **[2]**	
	Total for Question 4	**12**	

Qu.	Solution	Mark	Notes
5(a)	$F = \frac{74 \times 64}{253}$ $= 18.72$	 B1	or by adding row and subtracting from 74 or adding column and subtracting from 64.
	$G = \frac{(8 - 14.33)^2}{14.33}$ $= 2.80$	M1 A1	
	$H = 8.33 + 0.73 + G + 3.88 + 0.02$ $= 15.76$	 A1 **[4]**	FT their G
(b)	H_0: Attitude towards Welsh is independent of how Welsh was learned. H_1: Attitude towards Welsh is not independent of how Welsh was learned.	B1	both OR there is (not) association …
	$\chi^2 = 20.70 + 3.52 + H$ $\chi^2 = 39.98$	B1	FT their H, or using an appropriate contribution in the comparison with CV
	DF = 8 5% CV = 15.507	B1 B1	
	Since $39.98 > 15.507$ there is sufficient evidence to reject H_0.	M1	
	There is evidence to suggest that how a student learns Welsh affects their attitude towards the language.	A1	cso
(c)	Two different valid comments e.g. Students who learned Welsh from two parents are more likely to have a positive attitude towards the language. e.g. There is a real mix of attitudes towards the Welsh language amongst students who learned Welsh at school. e.g. There are far fewer than expected number of students that have a very negative attitude towards Welsh who learnt from two parents/carers.	E1×2 **[8]**	Must comment on more than just size of contributions.
	Total for Question 5	**12**	

<table>
<tr><th>Qu.</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>6</td><td>Identifying 140, 200, 260, 320 as the values of Y.</td><td>B1</td><td>B1 allow one error.</td></tr>
<tr><td></td><td><table><tr><td>y</td><td>140</td><td>200</td><td>260</td><td>320</td></tr><tr><td>$\mathrm{P}(Y=y)$</td><td>0.3</td><td>$2p$</td><td>p</td><td>$0.7 - 3p$</td></tr></table></td><td>B1

B1</td><td>Table may use rv$X$.
B1 for attaching $2p$ and p to 200 and 260 (or 6 and 7)
B1 for $0.7 - 3p$.</td></tr>
<tr><td></td><td>$\mathrm{E}(Y) = 0.3 \times 140 + 2p \times 200 + p \times 260 + (0.7 - 3p) \times 320$</td><td>M1</td><td>Using $\sum x\mathrm{P}(X = x)$ to form an expression in p (or another variable)</td></tr>
<tr><td></td><td>$206 = 42 + 400p + 260p + 224 - 960p$</td><td>M1</td><td>Set =206 (or 6.1)</td></tr>
<tr><td></td><td>$p = 0.2$</td><td>A1</td><td></td></tr>
<tr><td></td><td><u>ALTERNATIVE SOLUTION</u>
SIMULTANEOUS EQUATIONS</td><td></td><td></td></tr>
<tr><td></td><td><table><tr><td>y</td><td>140</td><td>200</td><td>260</td><td>320</td></tr><tr><td>$\mathrm{P}(Y=y)$</td><td>0.3</td><td>$2p$</td><td>p</td><td>q</td></tr></table></td><td>(B1)</td><td>B1 for attaching $2p$ and p to 200 and 260 (or 6 and 7) and labelling q.</td></tr>
<tr><td></td><td>$0.3 + 3p + q = 1$
$3p + q = 0.7$</td><td>(B1)</td><td>Either</td></tr>
<tr><td></td><td>$\mathrm{E}(Y) = 0.3 \times 140 + 2p \times 200 + p \times 260 + q \times 320$</td><td>(M1)</td><td>Using $\sum x\mathrm{P}(X = x)$ to form an expression in p and q</td></tr>
<tr><td></td><td>$206 = 42 + 400p + 260p + 320q$</td><td>(M1)</td><td>Set =206 (or 6.1)</td></tr>
<tr><td></td><td>Solve simultaneous equations
$165p + 80q = 41$ and $3p + q = 0.7$</td><td></td><td></td></tr>
<tr><td></td><td>$p = 0.2$ or $q = 0.1$</td><td>(A1)</td><td>A1 for either</td></tr>
<tr><td></td><td><table><tr><td>y</td><td>140</td><td>200</td><td>260</td><td>320</td></tr><tr><td>$\mathrm{P}(Y=y)$</td><td>0.3</td><td>0.4</td><td>0.2</td><td>0.1</td></tr></table></td><td>A1</td><td>Fully correct probability distribution, cao</td></tr>
<tr><td></td><td></td><td>[7]</td><td></td></tr>
<tr><td></td><td>Total for Question 6</td><td>7</td><td></td></tr>
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